

Problem 2

Find numbers a and b such that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} = 1$.

Solution

Evaluate the limit.

$$\begin{aligned} 1 &= \lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} \times \frac{\sqrt{ax+b}+2}{\sqrt{ax+b}+2} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{ax+b}-2)(\sqrt{ax+b}+2)}{x(\sqrt{ax+b}+2)} \\ &= \lim_{x \rightarrow 0} \frac{(ax+b)-4}{x(\sqrt{ax+b}+2)} \\ &= \lim_{x \rightarrow 0} \frac{ax+b-4}{x(\sqrt{ax+b}+2)} \end{aligned}$$

Set $b = 4$ so that there's only ax in the numerator. This allows x to cancel out.

$$\begin{aligned} 1 &= \lim_{x \rightarrow 0} \frac{ax}{x(\sqrt{ax+4}+2)} \\ &= \lim_{x \rightarrow 0} \frac{a}{\sqrt{ax+4}+2} \\ &= \frac{a}{\sqrt{a(0)+4}+2} \\ &= \frac{a}{4} \end{aligned}$$

Multiply both sides by 4 to obtain a .

$$a = 4$$